ATTACHMENT G-1: WAACY MAGNITUDE PDF MANUSCRIPT BY WOODDELL AND OTHERS

Alternative Magnitude-Frequency Distribution for Evaluating the Hazard of Multi-Segment Ruptures

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Abstract

The magnitude-frequency distributions (MFDs) commonly used for fault-specific sources in probabilistic seismic hazard analysis (PSHA), such as the exponential distribution, characteristic magnitude distribution of Youngs and Coppersmith (1985) or the maximum magnitude model of Hecker et al. (2011), specify the frequency of earthquakes up to a fixed upper limit that is typically defined based on the specification of a segmentation model for the fault.

Characterization of interacting faults with possible multiple segment linkages in less frequent very large ruptures requires specification of the relative frequency of each rupture scenario included in the hazard model that has been specified either by expert assessments coupled with seismic moment balancing (e.g. the Working Group on California Earthquake Probabilities, 2003) or by large-scale inversions of geological and geophysical data (e.g. UCERF3). To facilitate the incorporation of the concepts of potential fault segment linkages in standard PSHA studies we propose a generalized MFD that captures the characteristic magnitude behavior of the standard models while allowing for rare, large magnitude, linked ruptures. As described in Wooddell and Abrahamson (2013) the shape of the MDF is based on forward modeling of the coefficient of variation for slip at a point that is consistent with the global dataset of Hecker et al. (2013). Utilizing the new PDF and our simplified moment balance approach, we explore the hazard implications of this model and sensitivity to the selection of magnitude-displacement relationships, models for the probability of surface rupture, and other PDF model parameters.

Introduction

In the 1980s, seismic source characterization for probabilistic hazard studies began to use geologic information such as slip-rate and paleoseismic recurrence intervals to estimate the activity rate of faults rather than the traditional approach based on historical seismicity. If the activity rate is computed by balancing the moment rate on the fault with an exponential magnitude-frequency distribution (MFD) and the maximum magnitude is based on the fault length, then the resulting magnitude recurrence model tends to overpredict the rates of small earthquakes (Youngs and Coppersmith, 1985).

There are two ways that this over prediction of the small and moderate magnitudes can be avoided: (1) increase the maximum magnitude used in the expontential pdf or (2) maintain the maximum magnitude and change the MFD to have an increase in the rate of earthquakes near the maximum magnitude. Using earthquake observations on the Hayward fault, an example comparison of these approaches is shown in Figure 1.

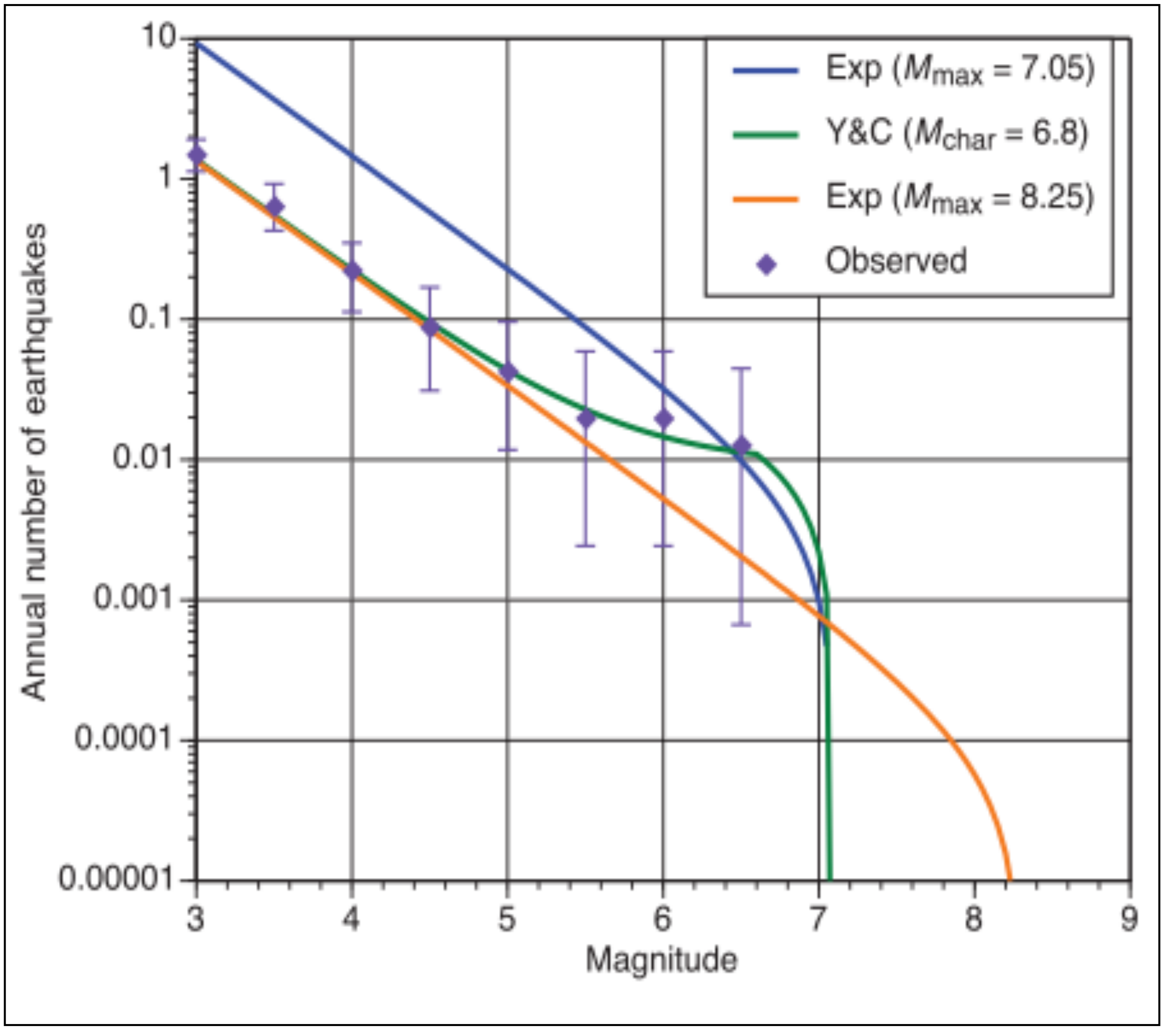

Figure 1. Comparison of the exponential model with M_{max} based on the fault length (blue), the Youngs & Coppersmith model that maintains M_{max} but places more momemt near the maximum magnitude (green), and the exponential model that increases M_{max} to fit the recurrence rates of the smaller magnitude earthquakes (orange). Purple diamonds are observations from the Hayward fault.

Hecker et al (2011) evaluated these two approaches for avoiding overprediction of the small and moderate magnitude earthquakes using the coefficient of variation (CV) of the slip at a point estimated from sites with measured slip from multiple earthquakes. They found that the CV from the slip data lead to a CV in the range of 0.40 to 0.55. They then computed the CV values of alternative magnitude – frequency distributions (Y&C1985 and exponential) utilizing a three-step forward modeling process consisting of: 1) computing a Probabilistic Rupture Hazard Analysis (PRHA), 2) using a Monte Carlo approach to simulate the distribution of surface displacements expected to be observed at a site from each of the MFDs, and 3) computing CV values from the simulated distributions to compare with the CV values from the empirical data. Comparison of the CV values from the forward modeling approach were compared with the CV values from the data to determine which MFDs are consistent with the data.

The 1985 Youngs and Coppersmith (Y&C1985) MFD lead to CV values that were consistent with the range from the earthquake observations; however, the exponential model, with a large maximum magnitude, lead to CV values that were greater than 0.55 (Figure 1). These results suggest that the Y&C1985 MFD is more consistent with geological observations of slip at a point on faults than the Gutenberg Richter MFD. Thus, based on this comparison, Hecker et al (2011) concluded that the exponential model with a large magnitude was not a viable model for the magnitude MFD on a single fault.

The Y&C1985 MFD, shown in Figure 2, has two parts: an exponential distribution for the smaller magnitudes and a uniform distribution (boxcar) for the characteristic magnitudes that is 0.5 magnitude units wide. The relative rates of the two parts of the distribution are defined by setting the height of the characteristic distribution at the same level as the exponential distribution one magnitude unit below the lower end of the boxcar. This approach to setting the relative rates of the two part of the distribution sounds arbitrary, but the key feature of this model is that it puts 94% of the seismic moment in the characteristic part and 6% in the exponential part. This is in contrast to the exponential model that puts about 50% of the moment into earthquakes with M<Mmax-0.5. Thus, the Youngs and Coppersmith model reduces the moment rate for earthquakes with M<Mmax-0.5 by a factor of 8 as compared to the exponential model.

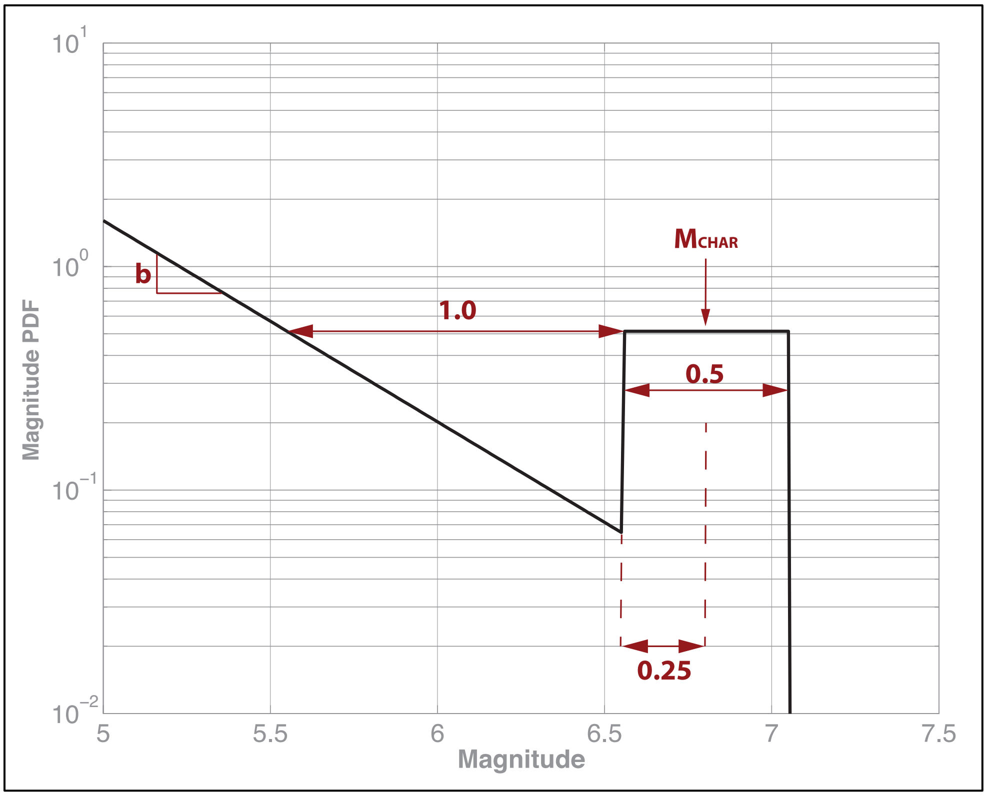

Figure 2. General form and parameters of the Youngs and Coppersmith (1985) MFD.

WAACY Model

One disadvantage of the Y&C1985 MFD is that, even though the it yields CV values more consistent with geological observation of slip at a point, it places a hard boundary at the mean characteristic magnitude +0.25 that does not allow for rare multi-fault or multi-segment ruptures. To allow for these larger ruptures, we develop an alternative form of the MFD called the WAACY model (for Wooddell, Abrahamson, Acevedo-Cabrera, and Youngs). In this model, the multi-segment ruptures are represented by a second exponential distribution for magnitudes above the characteristic magnitude range. In addition, the Y&C1985 MFD was generalized to use a normal distribution in place of the boxcar for the characteristic earthquake and to allow the user to set the relative rates of the lower exponential part with respect to the characteristic and the multi-segment parts based on the fraction of the total moment-rate that is released is in the lower exponential part.

The general form and parameters of the WAACY model are shown in Figure 3. The WAACY model has three parts to the MFD: a truncated exponential distribution for the lower magnitudes($M<M_1$), a truncated normal distribution for the characteristic part ($M_1 \leq M \leq M_2$), and a truncated exponential model for the multi-segment rupture part ($M>M_2$). The parameters needed to define the WAACY model are listed in Table 1, and the model is given by eq. (1), eq. (2), and eq. (3).

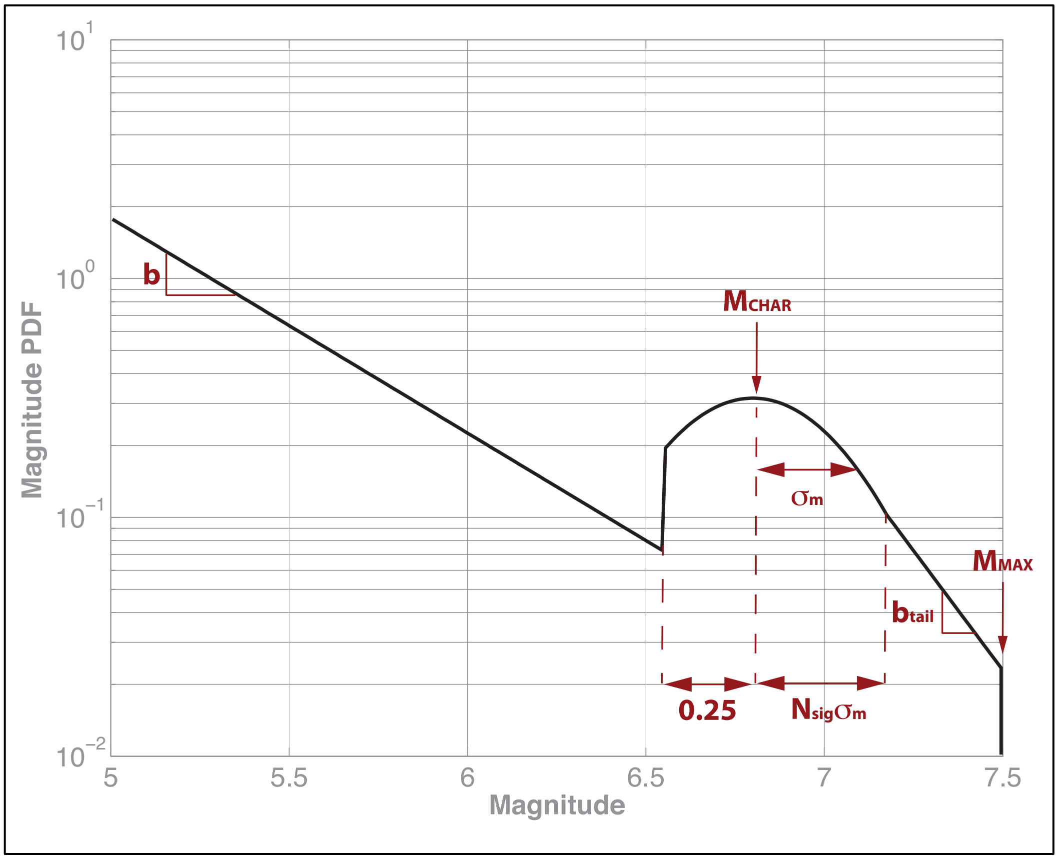

Figure 3. General form and parameters of the WAACY model.

Table 1. Parameters of the WAACY model.

Parameter	
b	b-value of the lower exponential part.
M_{char}	Mean characteristic magnitude
σ_m	Standard deviation of the characteristic magnitude
N_{sig}	Number of standard deviations for the transition from the characteristic part to the upper exponential part
M_{max}	Maximum magnitude
b_{tail}	b-value of the upper exponential part
F1	Fraction of the seismic moment that this released in the lower exponential tail
M1	Magnitude for the transition from the lower exponential part to the characteristic part $M1 = M_{char} - 0.25$
M2	Magnitude for the transition from the characteristic part to the upper exponential part $M2 = M_{char} + N_{sig}\,\sigma_m$

$$
MDF_{WAACY}(M) = \begin{cases} c_1 \dfrac{\beta \exp\left(-\beta(M - M_{\min})\right)}{1 - \exp\left(-\beta(M_1 - M_{\min})\right)} & \mathit{for}\ M < M_1 \\ c_2 \dfrac{1}{\sqrt{2\pi}\sigma_m} \exp\left(\dfrac{-(M - M_{char})^2}{2\sigma_m^2}\right) & \mathit{for}\ M_1 \le M \le M_2 \\ c_3 \dfrac{\beta_{tail} \exp\left(-\beta_{tail}(M - M_2)\right)}{1 - \exp\left(-\beta_{tail}(M_{\max} - M_2)\right)} & \mathit{for}\ M > M_2 \end{cases}
$$
eq. (1)

$$
c_4 = \frac{\beta_{tail}\sqrt{2\pi}\sigma_m}{\left(1 - \exp\left(-\beta_{tail}(M_{\max} - M_2)\right)\right)\exp\left(\dfrac{-(M_2 - M_{char})^2}{2\sigma_m^2}\right)} -
$$
eq. (2)

$$
c5 = \frac{F1\bar{M}_{o\_char}}{\bar{M}_{o\_char}}
$$
eq. (3)

CONSTRAINTS on WAACY

Using constraints on the CV of slip at a point to develop constraints on the MFD depends on the relation between slip at a point and earthquake magnitude. If there is strong scaling of slip with magnitude, then the CV of slip at a point provides a strong constraint on the variability of the magnitudes; however, if there is weak scaling of slip with magnitude, then the CV of slip at a point does not provide much of a constraint on the variability of magnitudes.

We use three different models for the scaling between average surface slip and magnitude: Wells and Coppersmith, 1994 (WC94); Hecker et al, 2013 (HEA13), and Shaw, 2013 (S13). The relations between the median slip and magnitude from these three models are given in eq. (4), eq. (5), and eq. (6).

$$
Log_{10}\left(D_{WC94}(M)\right) = -4.80 + 0.69M
$$
eq. (4)

$$
Log_{10}\left(D_{HEA13}(M)\right) = -2.79 + 0.41M
$$
eq. (5)

$$Log_{10}\left(D_{S13}(M)\right)=\begin{cases} -0.057+0.51(M-6.5) & for\, M<6.5 \\ -0.057+0.59(M-6.5)-0.133(M-6.5)^2 & for\, 6.5 \le M \end{cases}$$ eq. (6)

The three models are plotted in Figure 4. The WC94 model as the steepest slope and therefore, provides the strongest constraints on the MDF. The HEA13 model has the smallest slope for moderate magnitudes (M5 - M6.5) and has an intermediate slope for M>7. The S13 model has curvature to the magnitude scaling with an intermediate steep slope for moderate magnitudes, a steep slope in the M6.5 to M7 range, and a small slope in the M>7.5 range. With these different slopes, the three models will provide different constraints on the WAACY MFD when combined with the constraint on CV of slip at a point as shown in Figures 5a, 5b, and 5c for the WC94, HEA13, and S13 models respectively.

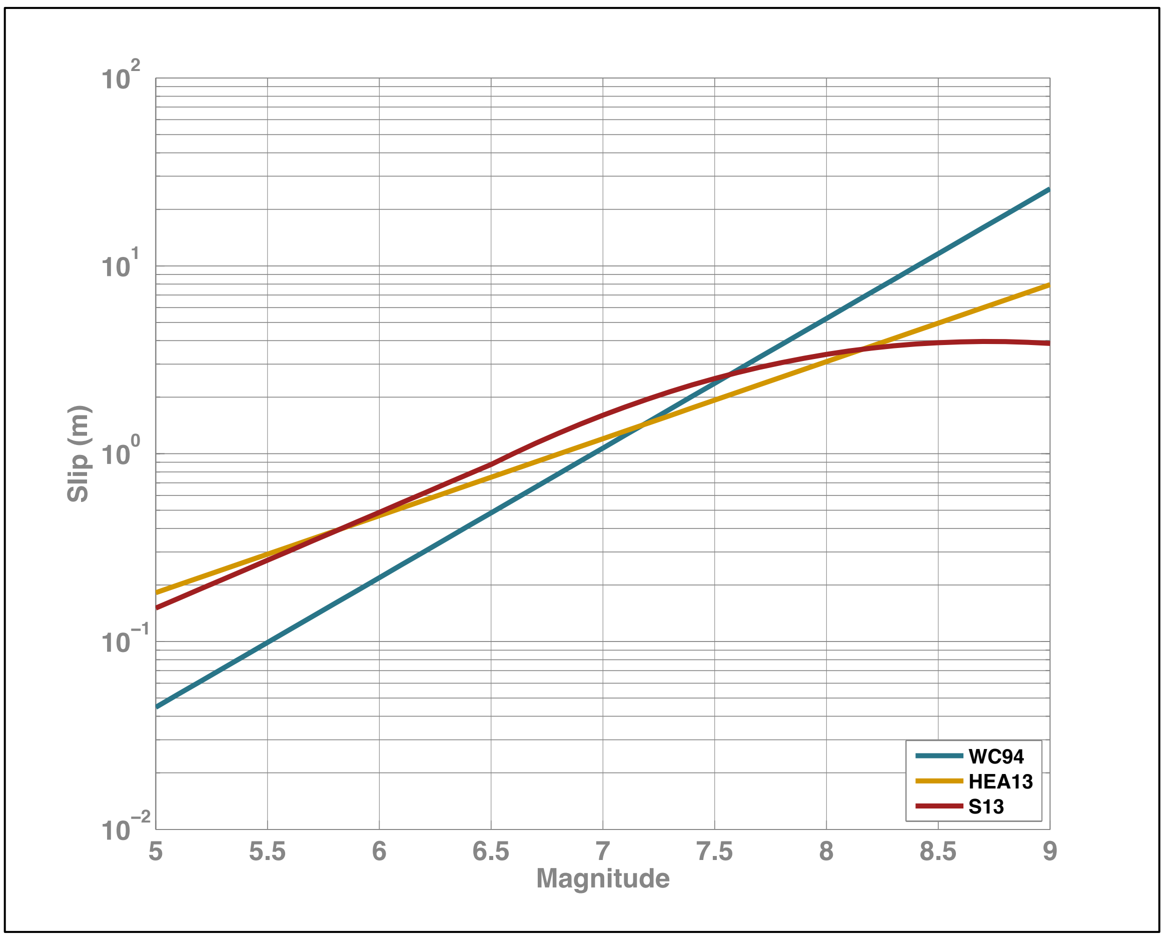

Figure 4. Comparison of three models for the scaling of magnitude with average surface slip: WC94 (eq. 4), HEA13 (eq.5), and S13 (eq. 6).

Figure 5 shows CV values computed from each of the three magnitude – displacement scaling relations. The CV ranges from Hecker et al. (2013) study are shown with the dashed lines. Therefore CV values that are consistent with Hecker et al. (2013) plot between the dashed lines.

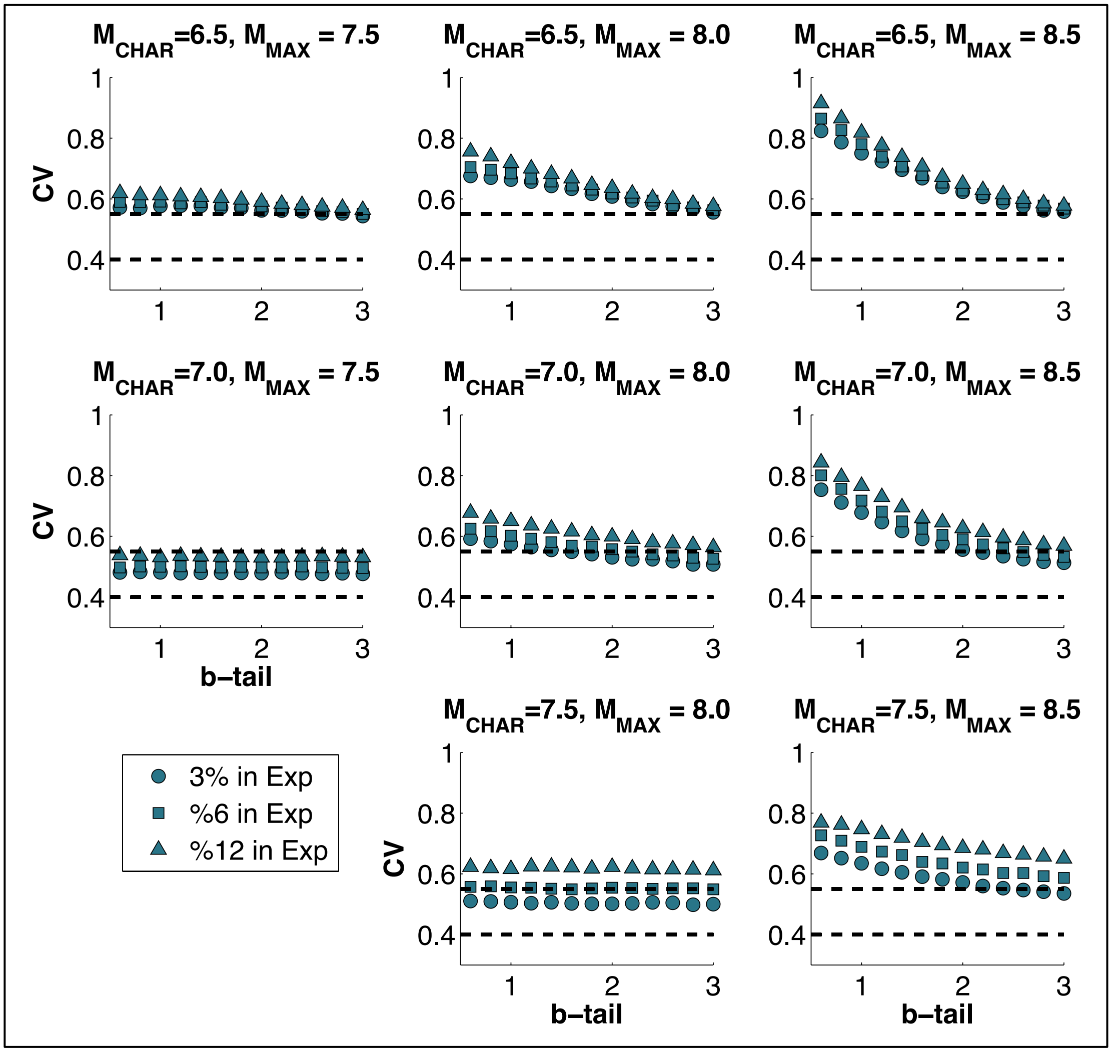

Figure 5a. Results for WC94 magnitude – displacement scaling. The black dashed lines show the range of acceptable CV values based on Hecker et al. (2013).

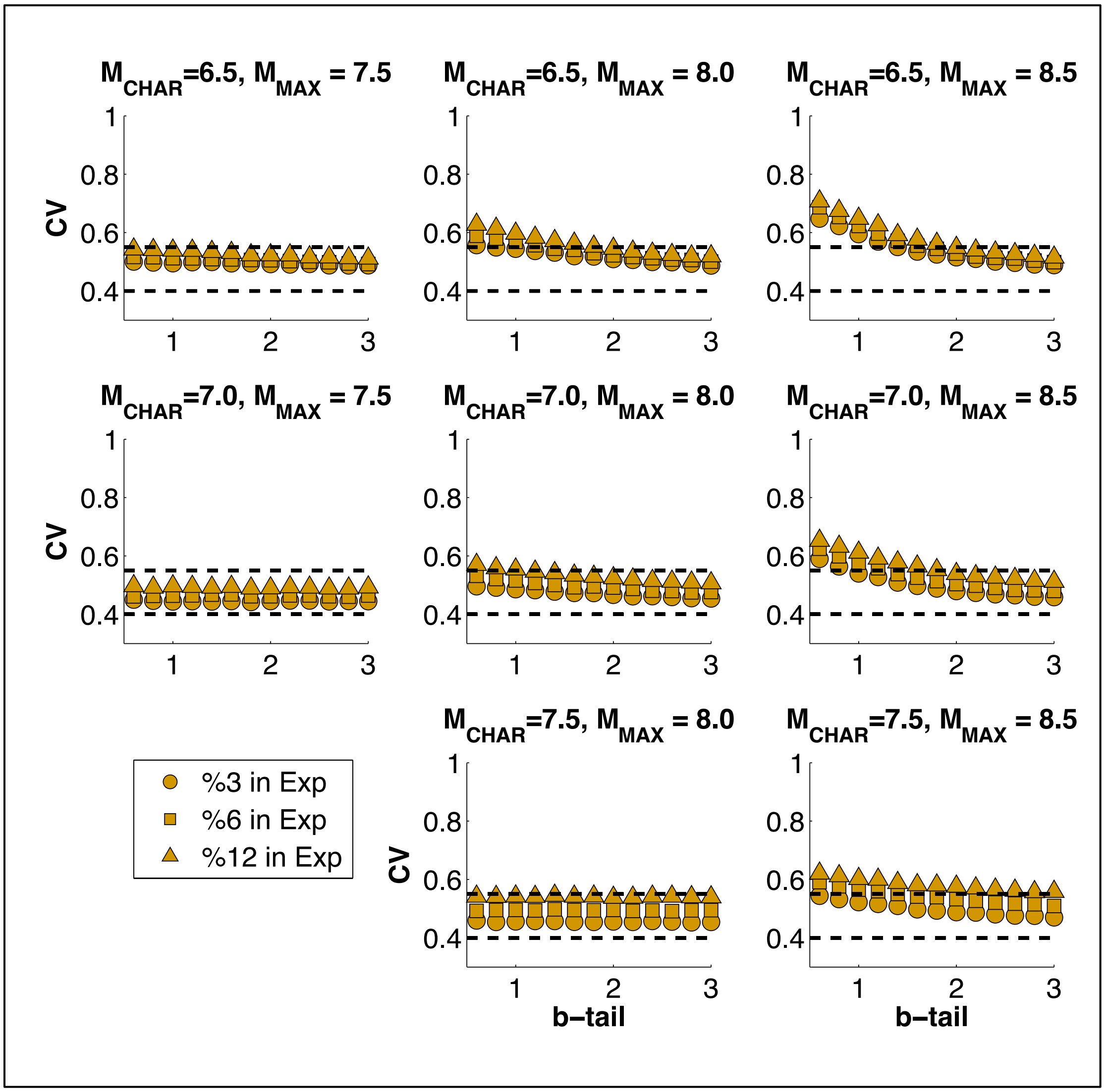

Figure 5b. Results for HEA13 magnitude – displacement scaling. The black dashed lines show the range of acceptable CV values based on Hecker et al. (2013).

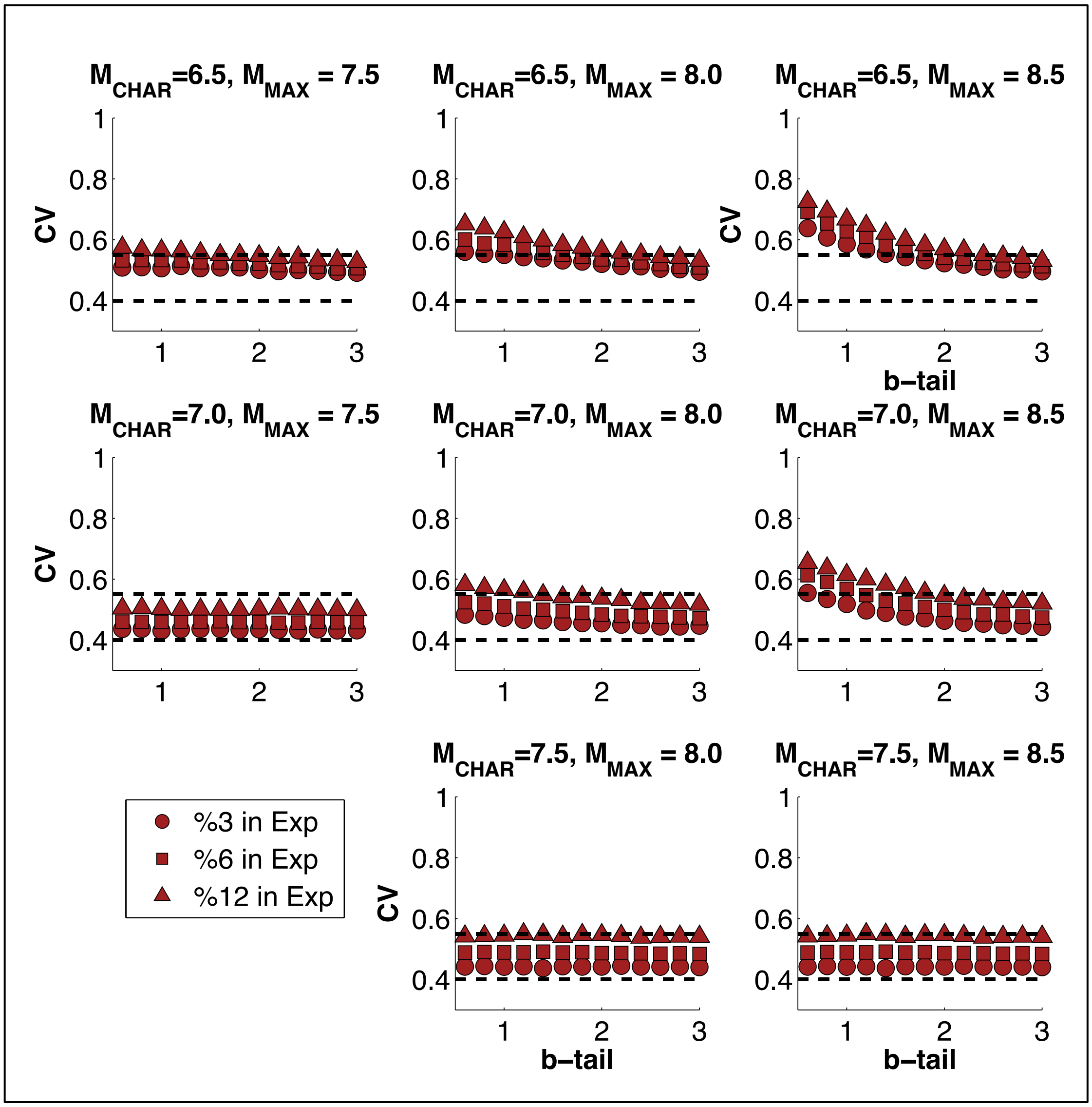

Figure 5c. Results for S13 magnitude – displacement scaling. The black dashed lines show the range of acceptable CV values based on Hecker et al. (2013).

Conclusions

In the analysis of the WAACY model, sensitivity to the following parameters was tested: percent of moment in the small to moderate magnitude exponential model, the value of b-tail, and the interval between M_{CHAR} and M_{MAX} for various magnitudes. A general conclusion from the sensitivity analyses, that holds for each of the tested magnitude – displacement models, is that as the distance between M_{CHAR} and M_{MAX} is decreased, the model becomes less sensitive to the value of b-tail. Therefore, b-tail has the most influence on the CV when the distance between M_{CHAR} and M_{MAX} becomes large. (Figures 5a, 5b, and 5c). Another trend that each of the models shows is that as the percent of the moment partitioned into the small to moderate magnitude exponential part increases, the CV increases proportionally.

Because of the relatively steep slope of the WC94 model (Figure 4), it provides the strongest constraints on the WACCY MFD. The S13 model, on the other hand, provides the least constraint because in our magnitude range of interest, above about M6.5, the slope of the S13 model decreases until it is flat at approximately M8. When the magnitude – displacement model is flat, there is no constraint on the CV.

Based on the above analysis, the WACCY model is generally consistent with the CV range of Hecker et al. when either the interval between M_{CHAR} and M_{MAX} is small, or the b-tail is high.